Cottenham Primary School Calculation Policy

2016-17

Contents

1.1 Understanding the Operation

Subtraction can either be seen as:

Take away – where the small number is subtracted or taken away from the larger number.

e.g. 78 – 12 =

392 – 171 =

4.68 – 1.24 =

Find the difference – counting up or on from the smaller number to the bigger number.

A secure understanding of the concept of inverse is fundamental to carrying out such calculations.

e.g. 22 – 18 =

101 – 97 =

267 – 299 =

5.5 – 1.8 =

Missing number problems can also be solved in a similar way

e.g. 28 – [ ] = 19 501 - [ ] = 192 [ ] - 43 = 82

Some calculations neatly fit into one category and the most efficient strategy would either be to take away or to find the difference by counting on. However, other calculations could be solved in either way.

It is important that children are taught to look carefully at the numbers involved when making their decision and that sufficient time is given within lessons for this. Choosing the correct method should be taught alongside the discrete methods themselves.

Efficiency is developed through progression in mathematical understanding and therefore children will have different 'efficient' strategies to one another at different times.

Take away calculations are often more appropriate when

- the digits of the smaller number are smaller than the digits of the larger number.

E.g. 86 – 23 =

4889 – 3426 =

- there is a large difference between the numbers

E.g. 95 – 3 =

7.15 – 0.03 =

Find the difference calculations are often more appropriate when

- the numbers are closer together

e.g. 163 – 148 =

- the larger number is a multiple of 10 or a near multiple of 10

e.g. 700 - 152 =

- the calculation involves bridging tens

e.g. 643 – 591 =

- missing number calculations

e.g. 53 – [ ] = 47

Examples of calculations that could be solved either way

e.g. 78 – 43 =

298 – 172 =

7.38 – 4.12 =

1.2 Progression of Mental Methods

All mental methods **must** first be scaffolded with models and/or images (e.g. using equipment or number lines), then removed when the children have internalised the mental method securely.

Counting forwards and backwards

Most children should be secure in these strategies by the end of KS1:

Example calculations	**Possible counting strategy**
8 - 3	Count back in ones from 8
17 - 3	Count back in ones from 17
18 - 6	Count back in twos from 18
57 - 3	Count back in ones from 57
80 - 7	Count back in ones from 80, or use number facts to 10, or place value knowledge
72 - 50	Count back in tens from 72
90 - 27	Count back in tens and then in ones from 90

Most children should be secure in these strategies by the end of KS2:

Example calculations	Possible counting strategy
90 - 27	Count back in tens and then in ones from 90 (or counting up from 27 if children are secure in finding the difference)
87 - 23	Count back in tens and then in ones from 87
73 - 68	Count up from 68, counting 2 to 70 and then 3 to 73
124 - 47	Count up from 47, counting 3 to 50, 50 to 100, then 24 to 124
960 - 500	Count back in 100s from 960
3.8 – 0.6	Count back in tenths
5.61 – 4.8	Count on in tenths and hundredths

Reordering

Most children should be secure in these strategies by the end of KS2:

Example calculations	Possible reordering strategy
12 - 7 - 2	12 - 2 - 7
17 + 9 - 7	17 - 7 + 9
58 + 47 - 38	58 - 38 + 47
4.6 - 0.4 - 1.6	4.6 - 1.6 - 0.4
2.3 + 1.5 - 0.3	2.3 - 0.3 + 1.5

Partitioning

Note how **only one number is partitioned.**
Partition the first number when subtracting multiples of 10 (examples 1 and 3).
Partition the second number in all other calculations.

Example calculations	Possible partitioning and counting strategy
78 - 40	70 + 8 - 40 70 - 40, then add the 8
68 - 32	68 - 30 - 2
365 - 40	360 + 5 - 40 360 - 40, then add the 5 Or 300 + 65 - 40 65 - 40, then add the 300
4.7 - 3.5	4.7 - 3 - 0.5
276 - 153	276 - 100 - 50 - 3

Partitioning: Bridging through multiples of 10

Example calculations	Possible bridging strategy
12 - 7	12 - 2 - 5
43 - 6	43 - 3 - 3
24 - 19	Count up one from 19 to 20, then 4 to 24
90 - 27	Count up three from 27 to 30, then 60 to 90
84 - 58	Count up two from 58 to 60, then 20 to 80, then 4 to 84 (or just up 24 in one go from 60 to 84)
607 - 288	Count up 12 to 300, 300 to 600 then 7 to 607
6070 - 4987	Count up 13 to 5000, 1000 to 6000 then 70 to 6070
9.6 - 1.7	9.6 - 1 - 0.6 - 0.1, or count up 0.3 from 1.7 to 2, then 7.6 to 9.6

Adjusting

Ensure that children understand this is only efficient when the numbers are near to multiples of 10 - “nearly numbers”.

Most children should be secure in these strategies by the end of KS2:

Example calculations	Possible adjusting strategy
70 - 9	70 - 10 + 1
84 - 18	84 - 20 + 2
95 - 78	95 - 80 + 2
405 - 399	405 - 400 + 1
6.8 - 4.9	6.8 - 5.0 + 0.1

1.3 Learning Number Facts

To support children with their fluency of subtraction calculations, there are certain maths facts that are useful for them to be able to recall quickly.

It is important that time is given for children to learn these facts supported by models and images, rather than just 'tested' on them.

These facts are divided into year group sections to support teachers, however it is important that teachers do not assume that the previous years' number facts are secure.

Children should be able to derive and recall:

Year 1

- Number pairs with a total of 10, e.g. 3 + 7 or what to add to a single-digit number to make 10, e.g. 3 + ? = 10
- Addition facts for totals to at least 5, e.g. 2 + 3, 4 + 3
- Addition doubles for all numbers to at least 10, e.g. 8 + 8
- Addition and subtraction facts for all numbers up to at least 10, e.g. 3 + 4, 8 – 5

Year 2

- Number pairs with totals to 20
- All pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 + ? = 100
- What must be added to any two-digit number to make the next multiple of 10,
  e.g. 52 + ? = 60
- Addition doubles for all numbers to 20, e.g. 17 + 17 and multiples of 10 to 50, e.g. 40 + 40
- Addition and subtraction facts for all numbers to 20, e.g. 9 + 8, 17 – 9, drawing on knowledge of inverse operations

Year 3

- Sums and differences of multiples of 10, e.g. 50 + 80, 120 – 90
- Pairs of two-digit numbers with a total of 100, e.g. 32 + 68, or 32 + ? = 100
- Addition doubles for multiples of 10 to 100, e.g. 90 + 90

Year 4

- Sums and differences of pairs of multiples of 10, 100 or 1000
- Addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves
- What must be added to any three-digit number to make the next multiple of 100, e.g. 521 + ? = 600
- Pairs of fractions that total 1

Year 5

- Sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3
- Doubles and halves of decimals, e.g. half of 5.6, double 3.4
- What must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + ? = 5000
- What must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + ? = 8

Year 6

- Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. 650 + ? = 930, ? – 1.4 = 2.5
- What must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. 7.26 + ? = 8

1.4 Formal Written Methods

Mental subtraction methods must be taught alongside formal written methods. Models and images such as number lines can be used to support this (see relevant section). Following this, formal written methods begin with extended methods, then compacted, and may progress to alternative written methods once understanding is secure. It is expected that from year 4 onwards, children **decide which operations and methods to use and why**.

Formal written methods are particularly appropriate for when the lower number has digits in it that are larger than in the first number e.g. 966 - 138.

Children **should not** be using any kind of formal written method unless the calculation cannot be carried out mentally. E.g. 763 - 531 could be completed mentally as 763 - 500 - 30 - 1.

Children should also **not** be using any formal written methods if their number facts and mental methods are not secured (see relevant sections).

See the Progression in Resources/Make It, Draw It, Say It, Write It for guidance on scaffolding formal written methods with models and images.

Children should work from right to left.

If children make significant errors at any stage, go back to the previous stage.

Progression in written methods according to the new curriculum	
Year Group	**Written Method**
Year 3	subtract three digit numbers, using formal written methods of columnar subtraction where appropriate
Year 4	subtract four digit numbers, using formal written methods of columnar subtraction where appropriate
Year 5	subtract whole numbers with more than 4 digits using formal written methods of columnar subtraction where appropriate
Year 6	*No specific reference to a formal written method of subtraction*

Progression in Expanded Methods

Exchanging units
966 – 138 =

$$\begin{array}{rrrrrrl} & 900 & + & {}^{50}\cancel{60} & + & {}^{1}6 & \\ - & 100 & + & 30 & + & 8 & \\ \hline & 800 & + & 20 & + & 8 & = 828 \end{array}$$

Exchanging tens
966 – 175 =

$$\begin{array}{rrrrrrl} & {}^{800}\cancel{900} & + & {}^{1}60 & + & 6 & \\ - & 100 & + & 70 & + & 5 & \\ \hline & 700 & + & 90 & + & 1 & = 791 \end{array}$$

Exchanging tens and units
906 – 177 =

$$\begin{array}{rrrrrrl} & {}^{800}\cancel{900} & + & {}^{90\ \cancel{100}}\cancel{0} & + & {}^{1}6 & \\ - & 100 & + & 70 & + & 7 & \\ \hline & 700 & + & 20 & + & 9 & = 729 \end{array}$$

Exchanging thousands
4673 – 2942 =

$$\begin{array}{rrrrrrrrl} & {}^{3000}\cancel{4000} & + & {}^{1600}\cancel{600} & + & 70 & + & 3 & \\ - & 2000 & + & 900 & + & 40 & + & 2 & \\ \hline & 1000 & + & 700 & + & 30 & + & 1 & = 1731 \end{array}$$

Exchanging tenths

10.54 - 1.62 =

$$
\begin{array}{rrrrrr}
 & {}^{9}\cancel{10} & + & \overset{1.5}{\cancel{0.5}} & + & 0.04 \\
- & 1 & + & 0.6 & + & 0.02 \\
\hline
 & 8 & + & 0.9 & + & 0.02 = 8.92
\end{array}
$$

Exchanging hundredths

10.54 - 1.38 =

$$
\begin{array}{rrrrrr}
 & 10 & + & {}^{0.4}\cancel{0.5} & + & \overset{0.14}{\cancel{0.04}} \\
- & 1 & + & 0.3 & + & 0.08 \\
\hline
 & 9 & + & 0.1 & + & 0.06 = 9.16
\end{array}
$$

Exchanging tenths and hundredths

10.54 - 1.68 =

$$
\begin{array}{rrrrrr}
 & {}^{9}\cancel{10} & + & {}^{\cancel{0.4}}\overset{1.4}{\cancel{0.5}} & + & \overset{0.14}{\cancel{0.04}} \\
- & 1 & + & 0.6 & + & 0.08 \\
\hline
 & 8 & + & 0.8 & + & 0.06 = 8.86
\end{array}
$$

Progression in Compacted Methods

Exchanging units:
966 – 138 =

$$\begin{array}{rccc} & 9 & {}^{5}\cancel{6} & {}^{1}6 \\ - & 1 & 3 & 8 \\ \hline & 8 & 2 & 8 \end{array}$$

Exchanging tens:
966 – 175 =

$$\begin{array}{rccc} & {}^{8}\cancel{9} & {}^{1}6 & 6 \\ - & 1 & 7 & 5 \\ \hline & 7 & 9 & 1 \end{array}$$

Exchanging tens and units.
906 – 177 =

$$\begin{array}{rccc} & {}^{8}\cancel{9} & {}^{9}{}^{1}\cancel{0} & {}^{1}6 \\ - & 1 & 7 & 7 \\ \hline & 7 & 2 & 9 \end{array}$$

Exchanging hundreds:
4673 – 2942 =

$$\begin{array}{rcccc} & {}^{3}\cancel{4} & {}^{1}6 & 7 & 3 \\ - & 2 & 9 & 4 & 2 \\ \hline & 1 & 7 & 3 & 1 \end{array}$$

Exchanging hundreds, tens and units:
12731 - 1367 =

12731 – 1367 = 11364

$$\begin{array}{rccccc} & & & 6 & 12 & 11 \\ & 1 & 2 & \cancel{7} & \cancel{3} & \cancel{1} \\ - & & 1 & 3 & 6 & 7 \\ \hline & 1 & 1 & 3 & 6 & 4 \end{array}$$

In this example it has been necessary to exchange from the tens and the hundreds columns.

Exchanging tenths:
96.6 – 13.8 =

$$
\begin{array}{rcccc}
 & 9 & {}^{5}\cancel{6} & . & {}^{1}6 \\
- & 1 & 3 & . & 8 \\
\hline
 & 8 & 2 & . & 8
\end{array}
$$

Exchanging hundredths:
9.66 - 1.75 =

$$
\begin{array}{rcccc}
 & {}^{8}\cancel{9} & . & {}^{1}6 & 6 \\
- & 1 & . & 7 & 5 \\
\hline
 & 7 & . & 9 & 1
\end{array}
$$

Exchanging tenths and hundredths:
9.06 – 1.77 =

$$
\begin{array}{rcccc}
 & {}^{8}\cancel{9} & . & {}^{9}\cancel{{}^{1}0} & {}^{1}6 \\
- & 1 & . & 7 & 7 \\
\hline
 & 7 & . & 2 & 9
\end{array}
$$

Follow the same methods for working with thousandths.

Alternative Methods

Expanded column subtraction, using negative numbers:

This can be a useful intermediate step before compact addition if children are struggling with the concept of exchanging, or it can just be used as an alternative method for those comfortable with exchanging.

E.g. ‘Exchanging’ tens
966 – 175 =

$$
\begin{array}{rl}
 & 900 + 60 + 6 \\
- & 100 + 70 + 5 \\
\hline
 & 800 + -10 + 1 = 791
\end{array}
$$

1.5 Subtracting Fractions

This is a short guide to subtracting fractions. Further guidance on the teaching of fractions can be found elsewhere.

There are 3 simple steps to subtracting fractions

- Step 1. Make sure the denominator are the same.
- Step 2. Subtract the numerator. Put the answer over the same denominator.
- Step 3. Simplify the fraction (if needed).

<u>Example 1:</u>

Step 1. The denominators are already the same. Go straight to step 2.

$$\frac{3}{4} - \frac{1}{4}$$

Step 2. Subtract the top numbers and put the answer over the same denominator:

$$\frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4}$$

Step 3. Simplify the fraction:

$$\frac{2}{4} = \frac{1}{2}$$

Example 2: $$\frac{1}{2} - \frac{1}{6}$$

Step 1. The denominators are different. See image below which supports this. They need to be slices of pizza of the same size before you can continue with the calculation.

$^{1}/_{2}$ − $^{1}/_{6}$ = ?

To make the denominators the same, multiply the top and bottom of the first fraction ($^{1}/_{2}$) by **3** like this:

× 3

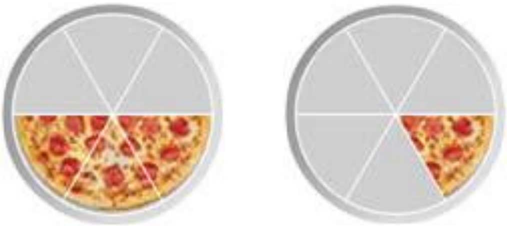
$$\frac{1}{2} = \frac{3}{6}$$

× 3

This is a significant jump for children to understand conceptually so do ensure children have a very secure understanding of fractions and equivalent fractions before teaching this. Children should also have an understanding of why this works.

And now the question looks like this:

$^{3}/_{6} \quad - \quad ^{1}/_{6}$

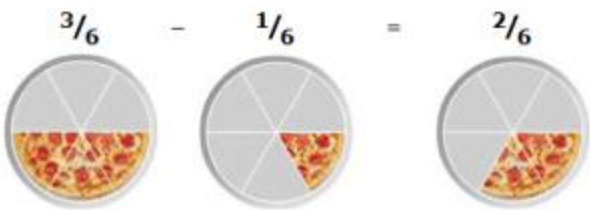


The denominators are the same, so we can go to step 2.

Step 2: Subtract the numerator and put the answer over the same denominator

$$\frac{3}{6} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6}$$

In picture form it looks like this:



Step 3. Simplify the fraction:

$$\frac{2}{6} = \frac{1}{3}$$

Appendices

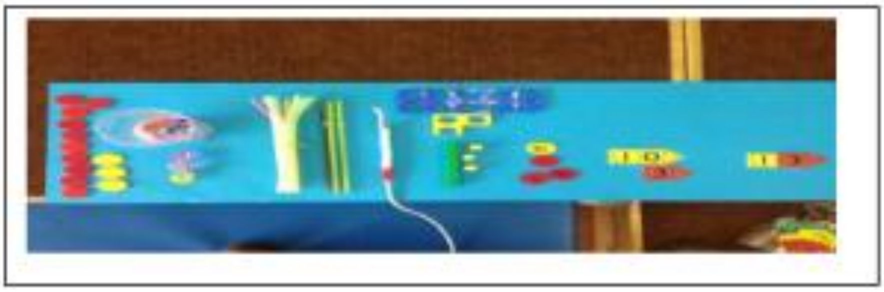
A1. Progression in Resources

There are three stages children go through in terms of developing their understanding through resources. It is essential that children throughout EYFS, KS1 and KS2 are exposed to all three stages when encountering resources in order to support their mathematical development.

Name of stage	Explanation	Examples of representation of 27	Examples of representation of 1.7
Enactive (Concrete)	Activity and movement often involved. Representations have no hidden meaning.	Learners make or draw models or images. E.g. To represent 27, a learner will count straws into 2 bundles of 10 and 7 single straws. Alternatively, they may put 10 counters into each of 2 pots, and 7 set out separately.	Learners make or draw models or images. E.g. To represent 1.7, a learner will take one strip of paper that represents a whole, and then cut a strip that represents one whole in to 10 parts, then count seven of them.
Iconic (Pictorial)	Representations may involve summaries. E.g. 10 is shown as a single item (Numicon or Dienes). At a more sophisticated level, 10 may be represented by a single counter (Place value counters)	Learners may use Numicon, Dienes, place value cards or place value counters to represent 27.	Learners use place value cards or counters to represent 1.7
Symbolic (Abstract)	Abstract recording which is dependent on an understanding of the symbols used.	Learners can place 27 on number lines, and write it in number sentences and calculations.	Learners can place 1.7 on number lines, and write it in number sentences and calculations.

Progression in the use of resources to support place value

This shows progression from iconic to symbolic resources.



NB: Learners of all ages, and **at all stages**, move between these three stages when developing understanding of mathematical concepts (or, indeed, any other area of skill or expertise). This means that **all children in all year groups** should have access to practical resources, and should be encouraged to use them to develop their own understanding, and to support their discussions and explanations of mathematical ideas.

A2. ‘Make it’, ‘Draw it’, ‘Say it’, ‘Write it’

This section develops the three stages of the Progression in Resources, using elements of the enactive, iconic and symbolic stage. In order for children to carry out calculations and understand what is involved, they need to be able to progress through the following stages of a calculation: ‘make it’, ‘draw it’, ‘say it’, ‘write it’. It is essential that the following are carried out in the correct order. Missing out one stage or starting at a later stage in the progression could result in misconceptions developing and children not fully understanding the calculation.

NB This approach is most effective when the children design and produce their *own* models and images to represent concepts. They should have ownership over the models and be the ones who are actively making, drawing etc. It is not something only the teacher models. However, at first, children may need some guidance in selecting the appropriate resources to use when “making it”.

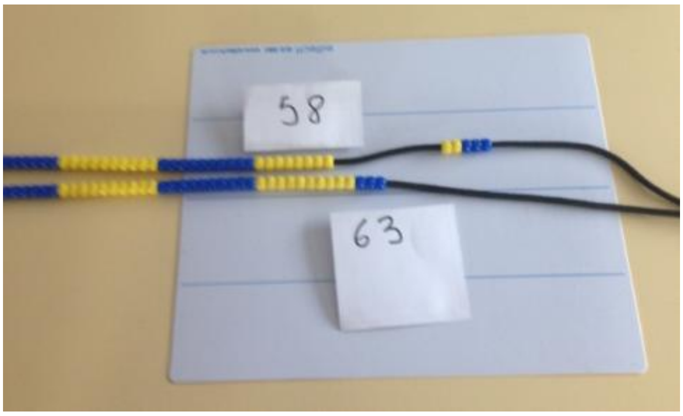
Example 1: 63 – 58 =

Children should not see this as a written number sentence until the ‘Write it’ stage.

Instead, give them written instructions ‘How many more is 63 than 58?’ or ‘What is the difference between 58 and 63?’

Make it: Get two bead strings, one showing 63 and the other showing 58.

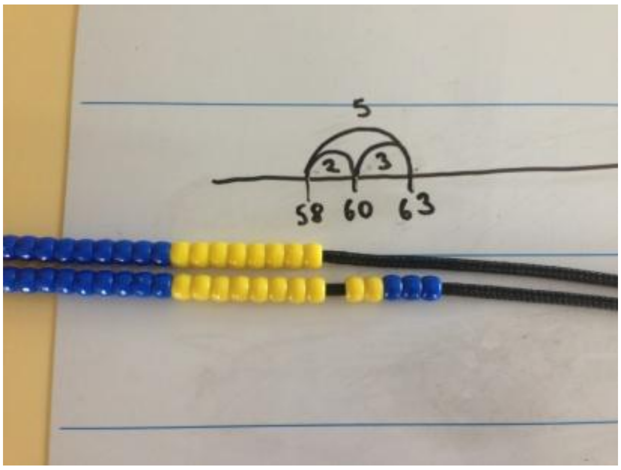
Line them up together.



This will help learners see that there is a small difference between these numbers and therefore the counting up method will work. The child could then count the beads that are the **difference** between the two numbers.

NB: it can be useful for more difficult concepts, such as finding the difference, to “make it” in several ways to embed the concept. For example, with two containers of water, two strips of paper etc. You could also show it on one strip of card or one bead string.

Draw it:



Children draw a number line, *alongside the bead-string,* thus bringing together making it and drawing it. Using the colours of the bead string they can see how you add on 2 to get to the next multiple of 10 and then add on 3 to get to 63. The making it very much feeds in to the drawing it.

Say it:

“The difference between 63 and 58 is 5.”

You can extend this for children by getting them to explain, such as “I know this because it takes 5 to count up from 58 to 63,” and explaining how they worked it out, step by step.

Write it:

Finally, the children write the calculation 63 – 58 = 5.

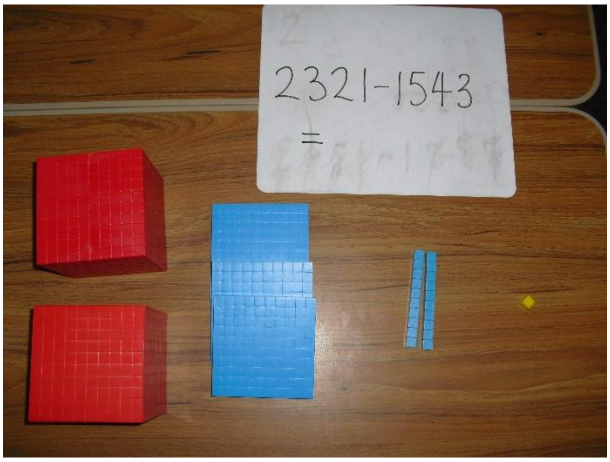
Note how many steps come **before** the children write these calculations. I.e. it is **not** the case that you provide children with a list of subtraction calculations and they then use the resources to help them work it out; using the resources very much exists as an initial stage on its own. Recording can be achieved through taking photographs.

Example 2 :

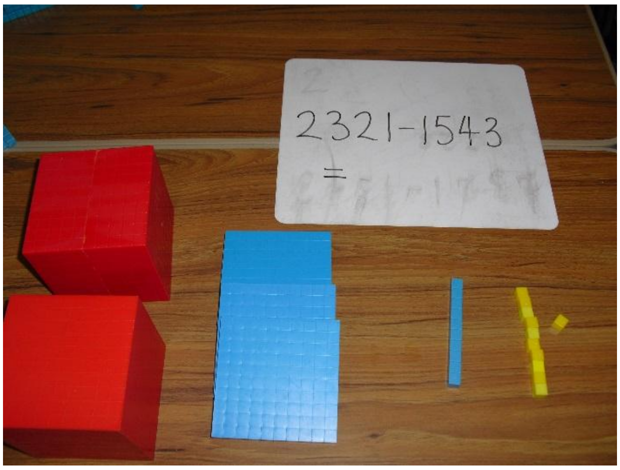
2321 -1543 =

Make it:

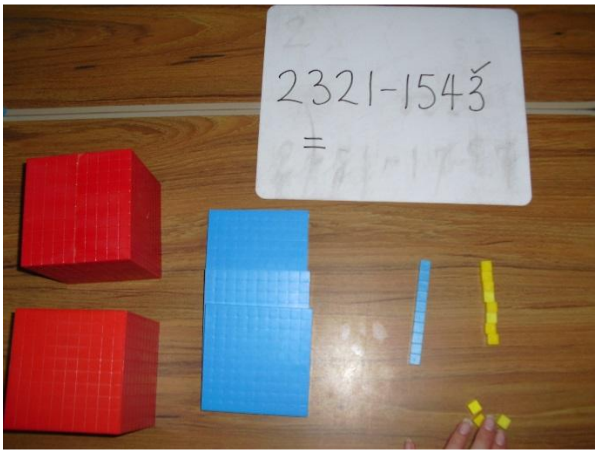
Make 2321 using dienes:



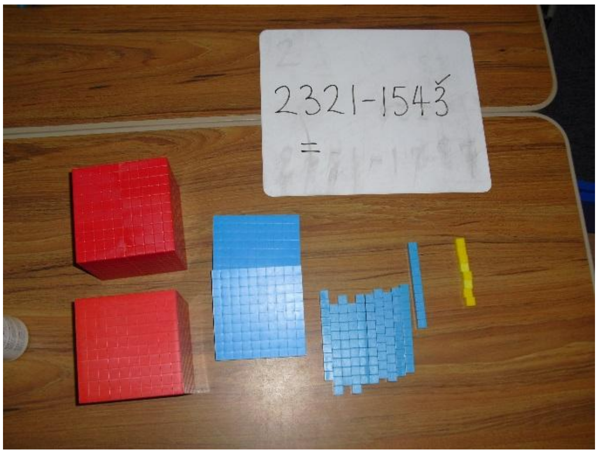
Children explore how to subtract 1543 by first taking away the units, which “cannot be done”. So they exchange a stick of ten for ten more units:

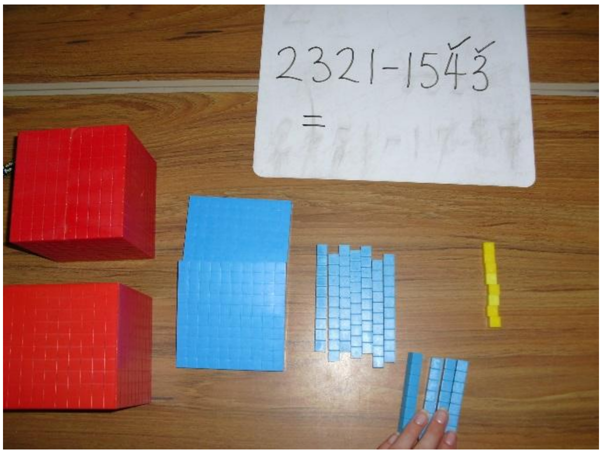


Now they can take the 3 units away:

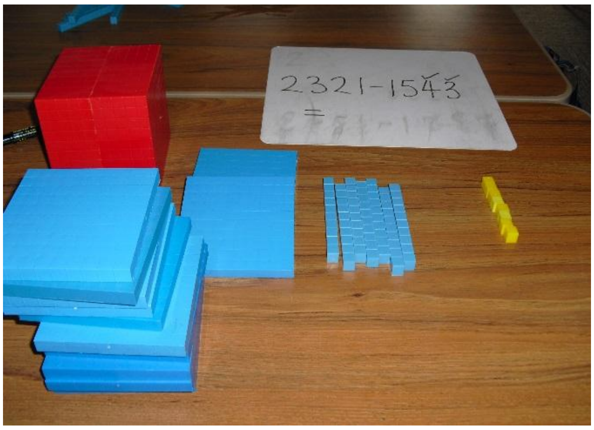


Children move on to taking away the tens. Children explore taking 4 tens away, but now realise this is impossible too. They exchange a hundred in order to get ten more tens. They can now take the 4 tens away.





Children move on to taking away the hundreds. Children explore taking 5 hundreds away, but now realise this is impossible too. They exchange a thousand in order to get ten more hundreds. They can now take the 5 hundreds away:



Finally, the children take away the thousands:

Draw It:

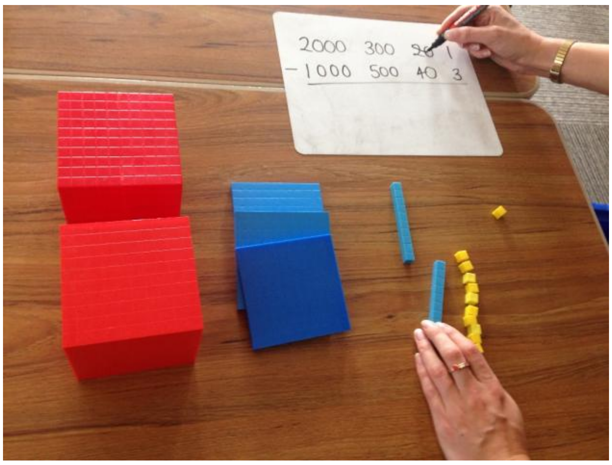
Children could then further embed this by using an example calculation to "draw it". For example, on sugar paper, they could come up with their own symbols to represent the dienes, and use this to explain the process to their peers (showing the exchanging and crossing out as they subtracted). This is a good opportunity for the teacher to check whether the "make it" stage has enabled the child to gain a deep understanding of the concept. If not, the child should go back to the "making it" stage, and they may need to work with different models.

Say It:

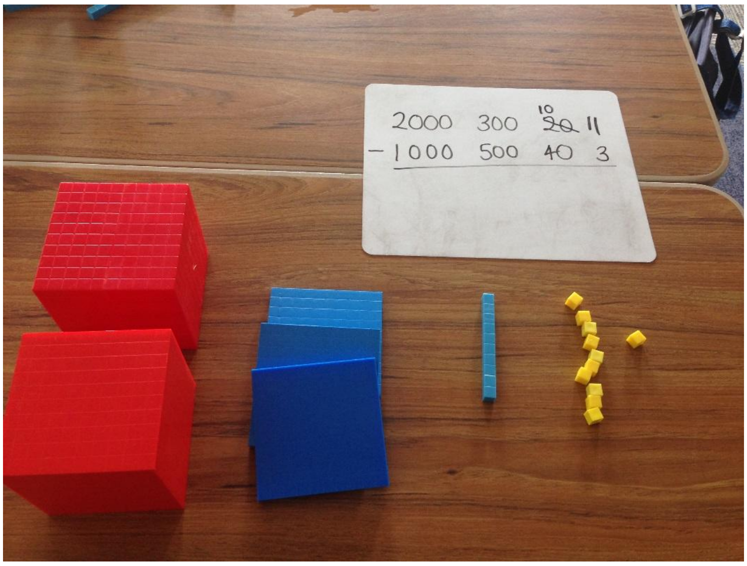
Children should be able to say what they are doing. E.g. Exchange a ten for ten units, then take the 3 units away. One way to explore this element in class could be for the children to pair up, and instruct the other in how to work through the steps of the calculation. This could also be done through discussing success criteria before starting, or the children instructing the teacher.

Write It:

When it comes to writing it, in this example it is useful to use that stage alongside the making it stage, for children to map the written method on to the model:



Here the learner has exchanged one ten for ten units, and then records this as she goes along:



NB: These examples are only guidelines for these calculations; you may feel that other models/images may be more suitable for your class.

A3. Number Lines

Number lines are **not** a written method of calculation. When used correctly, they are used as a scaffold; a supporting representation of number to aid children in their acquisition of mental methods. The number line starts off as being a detailed representation, and is then scaled back to become more and more abstract, until it is no longer required and the calculation can be done mentally.

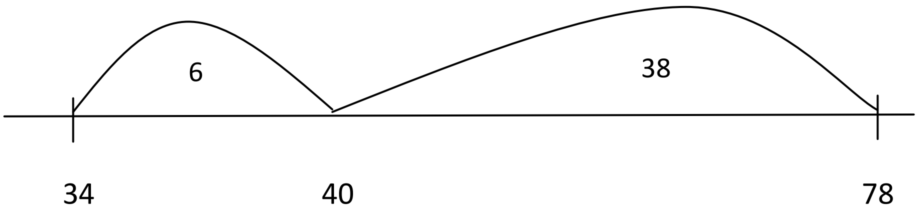
NB: Once children are secure in jumping in 10s, move them on to making bigger jumps on the number line. As a general rule of thumb, more than four jumps leaves too much of an opportunity for clerical errors.

Presentation

Number lines may exist as physical number tracks, plastic number lines, printed out number lines or drawn number lines.

Please ensure that all resources used follow these guidelines:
- the number line must start with the smaller number on the left
- the line must not have definite ends, instead it should extend to reinforce the concept of number as being continuous
- jumps on number lines must be on the top of the number line
- jumps must not have + or - signs in them

e.g.



Progression in use of number lines

e.g. 9 - 5 =

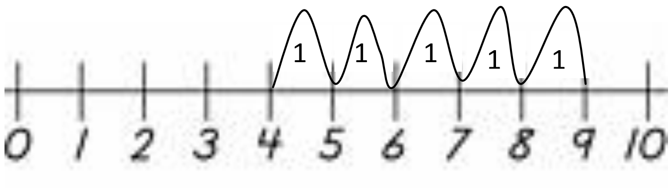
1. Children walk along the number track from 9. **Ensure children are saying aloud “8, 7, 6, 5, 4” instead of “1, 2, 3, 4, 5” because otherwise they are just counting to 5!**

2. Using a filled number track:

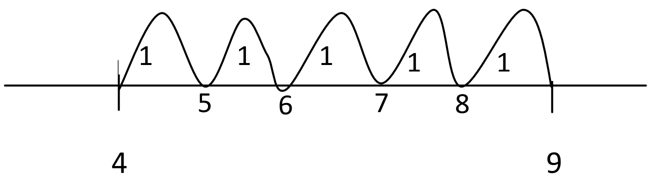
9 – 5 = 4

“Put your finger on 9 and count back 5, counting down in 1s.” **Again, the child needs to say aloud “8, 7, 6, 5, 4” rather than “1, 2, 3, 4, 5”.**

3. Using a filled number line + jottings:



4. Using an empty number line + jottings:

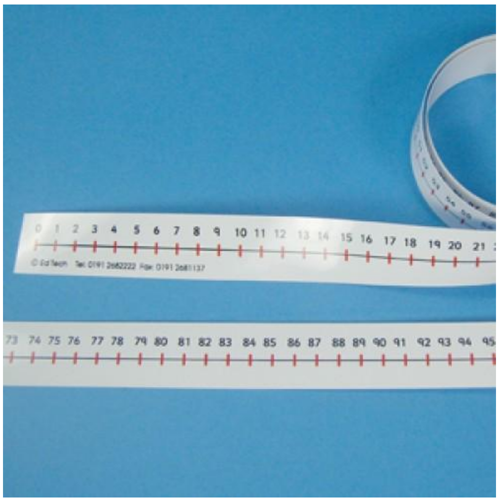


5. Not needing a number line, and jumping back 5 in one go mentally.

Number lines should also be used in key stage two in a similar way, but only in order to support children acquiring mental methods, not as a written method akin to columnar subtraction.

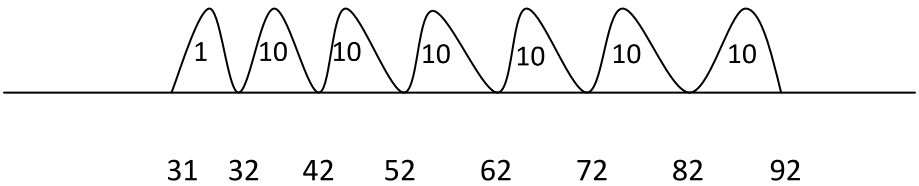
E.g. 92 - 61 and counting back.

1. Using a filled plastic number line:

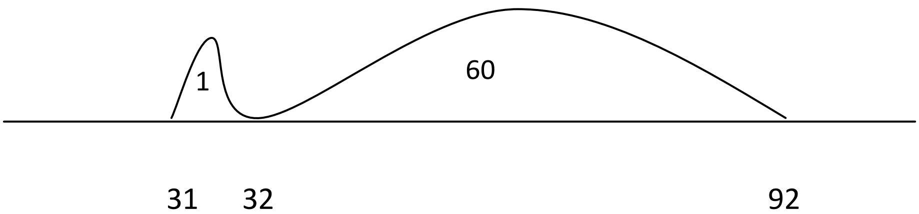


Circle 92 with a whiteboard pen, then jump back in tens, then in ones.

2. Using an empty number line + jottings:



3. Using an empty number line and jottings, and more efficient jumps:



A4. Vocabulary List

Use of "child-friendly" mathematical terms can be more appropriate for some children, particularly those with poor working memory.

Below is a list of the decided terms for use in Cottenham Primary School.

Child-friendly term	Definition
Friendly numbers	A multiple of 10, 100 or 1000
Nearly numbers	A number close to a multiple of 10, 100 or 1000

1.1. Understanding the operation

Multiplication

Three different ways of thinking about multiplication are:

- as repeated addition, for example 3 + 3 + 3 + 3
- as an array, for example four rows of three objects
- as a scaling factor, for example, making a line 3 cm long four times as long.

The use of the multiplication sign can cause difficulties. Strictly, 3 × 4 means four threes or 3 + 3 + 3 + 3. Read correctly, it means 3 multiplied by 4. However, colloquially it is read as '3 times 4', which is 4 + 4 + 4 or three fours. Fortunately, multiplication is commutative: 3 × 4 is equal to 4 × 3, so the outcome is the same. It is also a good idea to encourage children to think of any product either way round, as 3 × 4 or as 4 × 3, as this reduces the facts that they need to remember by half.

Division

Division can either be understood as either 'sharing' or 'grouping'.
Children will have more experience of sharing toys, biscuits etc and the concept of grouping will less familiar to them.

Sharing is when an amount of objects are shared equally between a set number of groups and the question is *How many in each group?*
e.g. for 12 ÷ 3 = 4
12 cubes are shared between 3 pots. How many cubes in each pot? 4 cubes are in each pot.

Grouping is when an amount of objects are put into predetermined groups and the question is *How many groups?*
e.g. for 12 ÷ 3 = 4
12 cubes are put into groups of 3. How many groups are there? There are 4 groups.

Interestingly the answer is the same both times, however it is important that children understand the difference and that sufficient time is spent on both concepts especially when 'real life' contexts are introduced.

Inverse
To have a secure understanding of multiplication and division, children will need to be aware of the relationship between the two and use the inverse of one to find answers to the other e.g. *I know that 134.1 ÷10 is 13.41 because I know that 13.41 x 10 is 134.1*
120 ÷ 12 = 10 because 12 x 10 = 120

It is important that multiplication and division are taught along side each other and reference is made to one whilst teaching the other rather than discrete lessons on each.

1.2 Number Facts

The National Curriculum makes clear that children should be able to derive and recall multiplication and division facts. If they cannot recall these facts rapidly and always resort to a basic counting strategy instead, they are distracted from thinking about the calculation strategy they are trying to use.

It is important that time is given for children to learn these facts supported by models and images, rather than just 'tested' on them. It is imperative that children can also apply these number facts to a range of contexts to include word problems and investigations.

For guidance in how to teach number facts and mental methods for multiplication – refer to the document Teaching Children to Calculate Mentally – Chapter 4 Multiplication and division strategies

1.3 Mental Methods:

There are four strategies that children can draw on when multiplying mentally.

- **Knowing multiplication and division facts for 12 x 12**
- Doubling and halving
- Multiplying and dividing by multiples of 10
- Multiplying and dividing by single digits numbers and multiplying by two digit numbers

Listed below are these number facts along with examples of appropriate mental methods divided into year group sections to support teachers, however it is important that teachers do not assume that the previous years' number facts are secure.

Children should be able to derive and recall:

EYFS

Derive doubles and halves to 10 through practical activities including songs, games, rhymes and discussions.

Year 1

Count on from and back to zero in ones, twos, fives
and tens
Recognise odd and even numbers to 20
Recall the doubles of all numbers to 10
Double all numbers to 10, e.g. double 9

Year 2

Count on from and back to zero in twos, threes, fives and tens.
Derive and recall multiplication facts for the 2, 5 and 10 times-tables and corresponding division facts up to the 12th multiple
Recognise odd and even numbers to 100
Recognise multiples of 2, 5 and 10
Double all numbers to 20 and find the corresponding halves, e.g. double 7, half of 14
Double multiples of 10 to 50, e.g. double 40, and find the corresponding halves
Double multiples of 5 to 50 and find the corresponding halves, e.g. double 35, half of 70

Year 3
Count on and back from zero in multiples of 4, 8, 50 and 100.
Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables up to 12th multiple
Derive and recall doubles of multiples of 10 to 100 and corresponding halves
Recognise multiples of 2, 3, 4, 5, 8 and 10 up to the 12th multiple
Double multiples of 10 to 100, e.g. double 90, and corresponding halves
Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170
Multiply one-digit and two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 × 100
Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence

Year 4
Count in multiples of 6, 7, 9, 25 and 1000
Identify doubles of two-digit numbers and corresponding halves
Derive doubles of multiples of 10 and 100 and corresponding halves
Derive and recall multiplication facts up to 12 x 12 and corresponding division facts
Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9,10, 11 and 12 up to the 12th multiple
Multiply by 0 and 1, divide by 1, multiplying three numbers together
Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94
Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680
Multiply numbers to 1000 by 10 and then 100, e.g. 325 × 10, 42 × 100
Divide numbers to 1000 by 10 and then 100 (whole-number answers),e.g. 120 ÷ 10, 600 ÷ 100, 850 ÷ 10
Multiply a multiple of 10 to 100 by a single-digit number, e.g. 60 × 3, 50 × 7
Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. 599 pence to £5.99, 2.5m to 250cm
Finding one quarter by finding one half
Multiply numbers to 20 by a single digit number

Year 5
Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
Recall squares of numbers to 12 × 12
Use multiplication facts to derive products of pairs of multiples of 10 and 100 and corresponding division facts
Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
Recall prime numbers up to 19
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
Recognise and use square numbers
Form equivalent calculations and use doubling and halving, e.g.
Multiply by 4 by doubling twice, e.g. 16 × 4 = 32 × 2 = 64
Multiply by 8 by doubling three times, e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96
Divide by 4 by halving twice, e.g. 104 ÷ 4 = 52 ÷ 2 = 26
Divide by 8 by halving three times, e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13
Multiply by 5 by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90
Multiply by 20 by doubling then multiplying by 10, e.g. 53 × 20 = 106 × 10 = 1060
Multiply by 50 by multiplying by 100 and halving
Multiply by 25 by multiplying by 100 and halving twice
Multiply and divide whole numbers and decimals by 10, 100 or 1000,e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10,

673 ÷ 100
Divide a multiple of 10 by a single-digit number (whole number answers), e.g. 80 ÷ 4, 270 ÷ 3
Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g. 60 × 30, 900 × 8
Multiply by 25 or 50, e.g. 48 × 25, 32 × 50 using equivalent calculations, e.g. 48 x 100 ÷ 4, 32 x 100 ÷ 2
Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm
Multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8
Multiply two-digit numbers by 5 or 20, e.g. 32 × 5, 14 × 20
Multiply by 25 or 50, e.g. 48 × 25, 32 × 50

Year 6

Refer to previous year groups to ensure all Number facts are secure
Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2
Identify common factors, common multiples and prime numbers
Perform mental calculations, including with mixed operations and large numbers.
Form equivalent calculations and use doubling and halving, e.g. divide by 25 by dividing by 100 then multiplying by 4e.g. 460 ÷ 25 = 4.6 × 4 = 18.
Divide by 50 by dividing by 100 then doubling e.g. 270 ÷ 50 = 2.7 × 2 = 5.4
Multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20
Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300
Divide by 25 or 50
Multiply a two-digit and a single-digit number, e.g. 28 × 7
Divide a two-digit number by a single-digit number e.g. 68 ÷ 4
Divide by 25 or 50, e.g. 480 × 25, 3200 × 50
Find new facts from given facts, e.g.
given that three oranges cost 24p, find the cost of four oranges

1.3 Formal Written Methods in Multiplication and Division

Mental strategies must be taught alongside formal written methods. Models and images such as arrays and number lines can be used to support this. See the Resources/Make It-Draw It-Say It-Write It for guidance on scaffolding these methods (using arrays, grids, number lines etc.).

Following this, formal written methods begin with short multiplication and division (typically around Year 4), followed by long multiplication and division (for most children during Year 5).

Children **should not** be using any kind of formal written method unless the calculation cannot be carried out mentally e.g.

13 x 8 could be completed mentally as 10 x 8 = 80 and 3 x 8 = 24, so 13 x 8 = 104 (80 + 24).

Children should be encouraged to simplify the calculation where possible using halving, doubling as well as known number facts.

e.g. 648 ÷ 24 is the same as 324 ÷ 12 (dividing both numbers by 2)

324 ÷ 12 is the same as 108 ÷ 4 (dividing both numbers by 3)

108 ÷ 4 is the same as 54 ÷ 2 (dividing both numbers by 2), which is 27.

When introducing formal written methods, if children make significant errors at any stage go back to the previous stage

Depth and Breadth of curriculum
Ensure children have depth and breadth of their year group's curriculum rather than moving onto the next year group's objectives. This can be achieved through opportunities to use and apply their learnt methods and facts in a range of contexts. Children should be taught to reason and problem solve fluently.

Progression in written *multiplication* methods according to the new curriculum	
Year Group	**Written Method**
Year 3	Pupils develop reliable written methods for multiplication…, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method[s] of short multiplication…
Year 4	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
Year 5	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.
Year 6	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Multiply simple pairs of proper fractions, writing the answer in its simplest form.

Progression in written *division* methods according to the new curriculum	
Year Group	**Written Method**
Year 3	Pupils develop reliable written methods for division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written method[s] of short division.
Year 4	Pupils practise to become fluent in the formal written method of short division with exact answers.
Year 5	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
Year 6	Divide numbers us to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpret remainders according to the context. Divide proper fractions by whole numbers. Associate a fraction with division and calculate decimal fraction equivalents e.g. 0.375 for a simple fraction e.g. 3/8

Short multiplication (expanded method)

To aid children's understanding, it is important to teach the expanded method of short multiplication first.

24 x 6 would be:

		2	4	
x			6	
		2	4	(6 x 4)
	1	2	0	(6 x 20)
	1	4	4	

342 x 7 would be:

		3	4	2	
x				7	
			1	4	(7 x 2)
		2	8	0	(7 x 40)
	2	1	0	0	(7 x 300)
	2	3	9	4	

2741 x 6 would be:

		2	7	4	1	
x					6	
					6	(6 x 1)
			2	4	0	(6 x 40)
		4	2	0	0	(6 x 700)
	1	2	0	0	0	(6 x 2000)
	1	6	4	4	6	

Once children are secure with the expanded method of short multiplication, the compact method should be taught.

24 × 6 becomes

		2	4
×			6
	1	4	4
		2	

Answer: 144

342 × 7 becomes

		3	4	2
×				7
	2	3	9	4
		2	1	

Answer: 2394

2741 × 6 becomes

		2	7	4	1
×					6
	1	6	4	4	6
		4	2		

Answer: 16 446

Long multiplication

Once secure with the concept of 'exchanging', children can be introduced to the long multiplication method, which will enable them to efficiently work with larger numbers.

124 × 26 becomes

	1	2	
	1	2	4
×		2	6
	7	4	4
2	4	8	0
3	2	2	4
1	1		

Answer: 3224

Short Division

To understand short division it is important to have mastered the skills of multiplication and subtraction.

98 ÷ 7 would be

$$\begin{array}{rl|lll}
 & & 1 & 4 & \\ \hline
7 & & 9 & 8 & \\
- & & \underline{7} & \underline{0} & (10 \times 7) \\
 & & 2 & 8 & \\
- & & \underline{2} & \underline{8} & (4 \times 7) \\
 & & R & 0 &
\end{array}$$

432 ÷ 5 would be

$$\begin{array}{rl|llll}
 & & 8 & 6 & r\,2 & \\ \hline
5 & & 4 & 3 & 2 & \\
- & & \underline{4} & \underline{0} & \underline{0} & (80 \times 5) \\
 & & & 3 & 2 & \\
- & & & \underline{3} & \underline{0} & (6 \times 5) \\
 & & & R & 2 &
\end{array}$$

496 ÷ 11 = would be

$$\begin{array}{rl|llll}
 & & 4 & 5 & r\,1 & \\ \hline
11 & & 4 & 9 & 6 & \\
- & & \underline{4} & \underline{4} & \underline{0} & (40 \times 11) \\
 & & & 5 & 6 & \\
- & & & \underline{5} & \underline{5} & (5 \times 11) \\
 & & & R & 1 &
\end{array}$$

98 ÷ 7 becomes

$$\begin{array}{r|ll}
 & 1 & 4 \\ \hline
7 & 9 & {}^{2}8
\end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r|llll}
 & & 8 & 6 & r\,2 \\ \hline
5 & 4 & {}^{3}3 & 2 &
\end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r|llll}
 & & 4 & 5 & r\,1 \\ \hline
1\;1 & 4 & 9 & {}^{5}6 &
\end{array}$$

Answer: $45\frac{1}{11}$

Long Division

432 ÷ 15 becomes

$$\begin{array}{r|llll}
 & & 2 & 8 & r\,12 \\ \hline
1\;5 & 4 & 3 & 2 & \\
 & 3 & 0 & 0 & \\ \hline
 & 1 & 3 & 2 & \\
 & 1 & 2 & 0 & \\ \hline
 & & 1 & 2 &
\end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r|llll}
 & & 2 & 8 & \\ \hline
1\;5 & 4 & 3 & 2 & \\
 & 3 & 0 & 0 & 15\times20 \\ \hline
 & 1 & 3 & 2 & \\
 & 1 & 2 & 0 & 15\times8 \\ \hline
 & & 1 & 2 &
\end{array}$$

$$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r|llll}
 & & 2 & 8\cdot & 8 \\ \hline
1\;5 & 4 & 3 & 2\cdot & 0 \\
 & 3 & 0 & \downarrow & \downarrow \\ \hline
 & 1 & 3 & 2 & \\
 & 1 & 2 & 0 & \\ \hline
 & & 1 & 2 & 0 \\
 & & 1 & 2 & 0 \\ \hline
 & & & & 0
\end{array}$$

Answer: 28·8

Multiplying Fractions

There are three simple steps to multiplying fractions. An example of the steps is shown below.

Step 1. Multiply the top numbers:

$$\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{} = \frac{9}{}$$

Step 2. Multiply the bottom numbers:

$$\frac{1}{3} \times \frac{9}{16} = \frac{1 \times 9}{3 \times 16} = \frac{9}{48}$$

Step 3. Simplify the fraction:

$$\frac{9}{48} = \frac{3}{16}$$

To multiply fractions by whole numbers, the steps are exactly the same. However, before completing the calculation, it is important to remember to make the whole number the numerator and to give it a denominator of 1. e.g.

$$\frac{3}{4} \times 2 \qquad \text{becomes} \qquad \frac{3}{4} \times \frac{2}{1}$$

This calculation can then be solved by following the three steps detailed above.

NB multiplying a whole number by a fraction or mixed number is a Year 5 objective and multiplying fractions together is a Year 6 objective.

Appendix 1

Progression in using a number line for multiplication and division

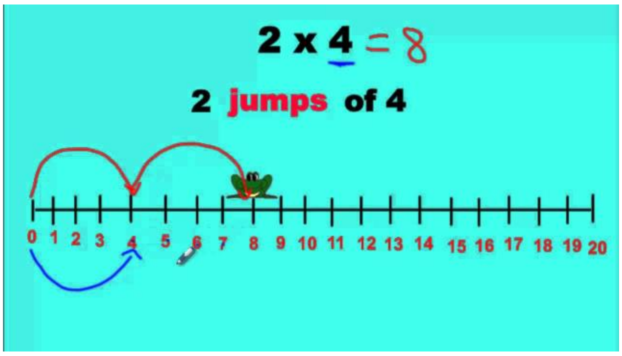
Number lines are not a formal written method. They are a mental method with jottings.

- Counting on in steps of a number

e.g. Counting on in 5s on a counting stick

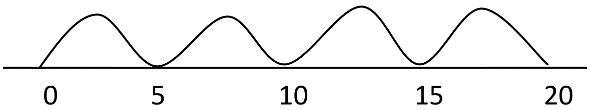
5 10 15 20 25 30 35 40 45 50

- Counting on in multiples on a filled number line



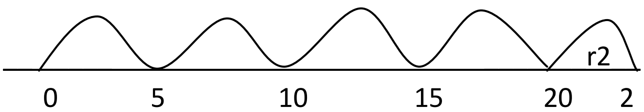
- ‘Counting on’ on an empty number line

4 x 5 = 20
20 ÷ 5 = 4



- Dividing with remainders using a number line

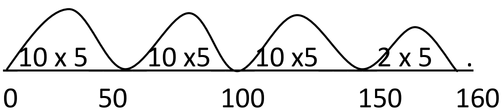
22 ÷ 5 = 4 r 2



- Using a ‘What I know’ box for multiplying 2 digit numbers and applying this to a number line

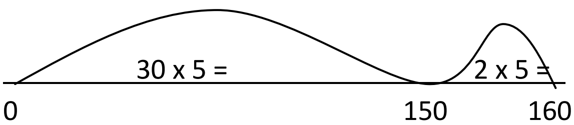
1 x 5 = 5
2 x 5 = 10
3 x 5 = 15
5 x 5 = 25
10 x 5 = 50

32 x 5 = 160



Using a number line with less steps:

32 x 5 = 160

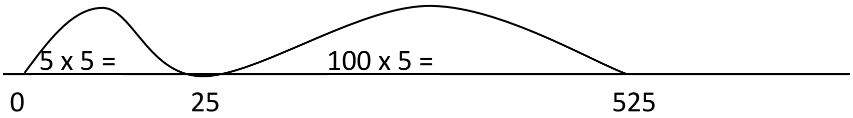


Using a number line to divide 3 digit numbers

525 ÷ 5 = 105

Create a 'what I know box'

1 x 5 = 5
2 x 5 =10
3 x 5 = 15
5 x 5 = 25
10 x 5 = 50



A2

Arrays to support multiplication and division

This document explains the progression from the 'concrete', 'pictorial' and 'abstract' with reference to arrays and the grid method for multiplication.

A secure grasp of arrays will support children with their understanding of the inverse relationship between multiplication and division.

Each array can be described using the language of both multiplication and division. However, at the point when the grid method is introduced, this is not an appropriate method for division calculations.

Part 1

Multiplication and Division

Arrays

Show children practical 'real life' images of arrays e.g. cake tins, ice cube trays, bars of chocolate

Say it - Use the vocabulary of 'rows of' and 'columns' of when describing the array.

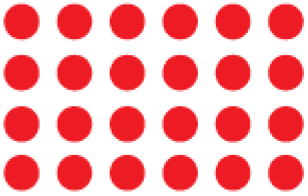
Describe the array as '12 cakes have been divided into rows of 3. How many in each row?' There are 4 cakes in each row.

Make it – Allow children to make their own arrays using cubes, counters etc

e.g. 4 rows of 4 toy cars.

16 cars have been divided into 4 rows of 4.

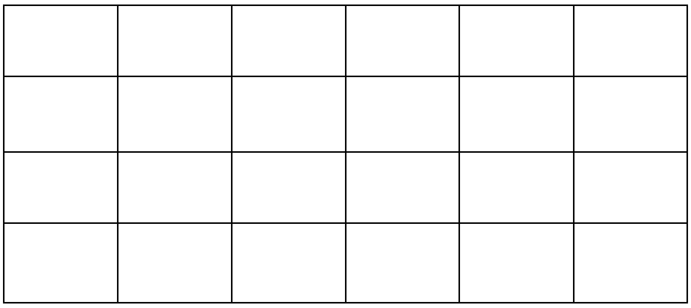
Draw it – involve children in drawing their own arrays



Say it: Describe the arrays in various ways

e.g. 4 + 4 + 4 + 4 + 4 + 4 = 24 6 columns of 4 4 rows of 6 6 groups of 4	4 x 6 = 24 24 ÷ 6 = 24 ÷ 4 =

An array can then be seen as a grid e.g. 6 x 4 = 24
The grid can also be described as 24 ÷ 6 = 4

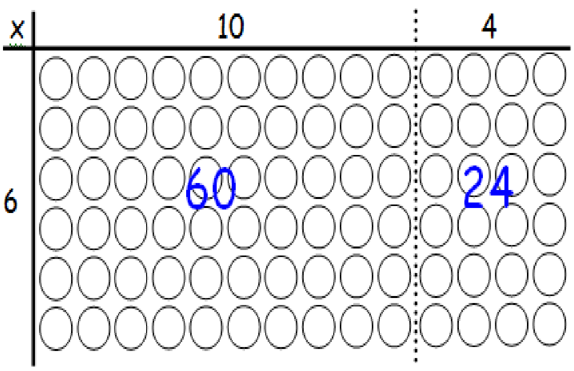


Part 2
Multiplication only

A grid as an array supports the grid method for multiplication but is not an appropriate strategy for division, therefore division examples have not been included.

It is really important for children to see the grid method presented as an array.

For example for the calculation:
14 x 6 =



14 x 6 – can also be seen as

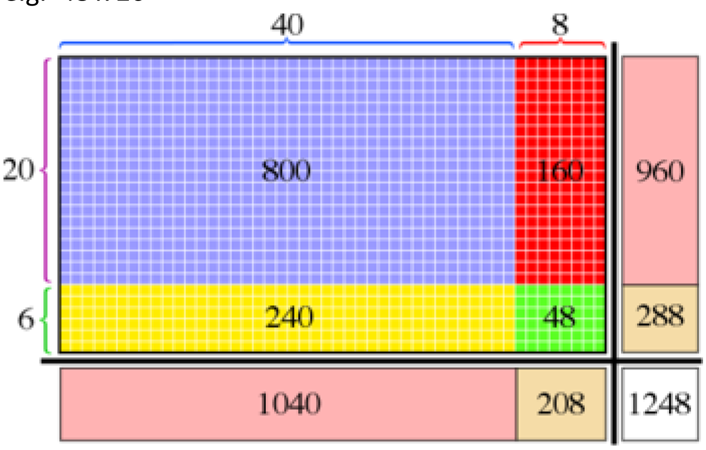
X	10	4
	OOOOOOOOOO	OOOO
6	O	
	O	
	O	
	O	
	O	

To help children see 10 x 6 = 60 and 6 x 4 = 24
Would lead onto:

x	10	4
6	60	24

60 + 24 = 84

This can then be applied to multiplying two digit numbers
e.g. 48 x 26

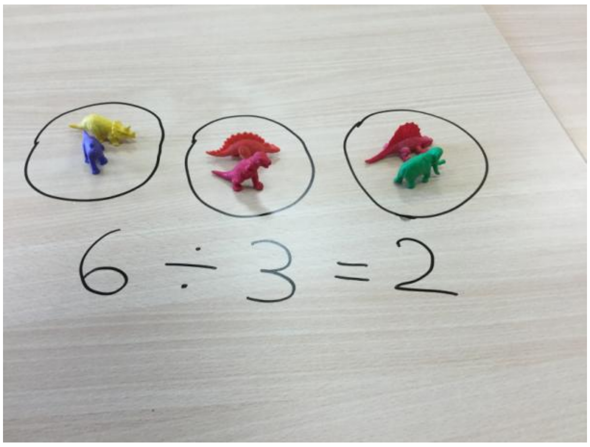


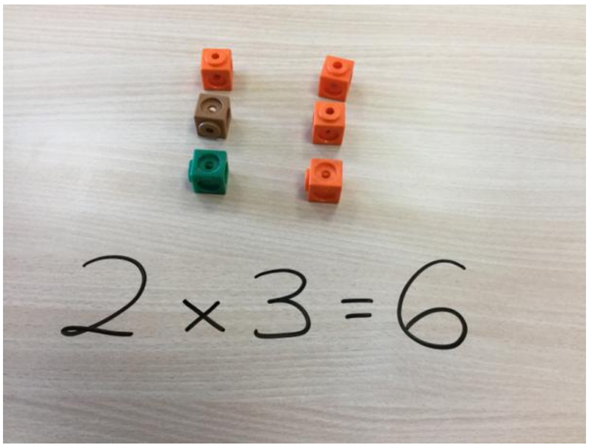
Which leads on to:

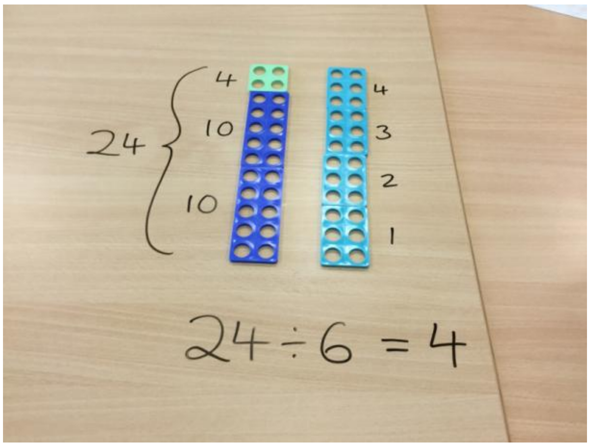
x	40	8	
20	800	160	960
6	240	48	288
1040		208	1248

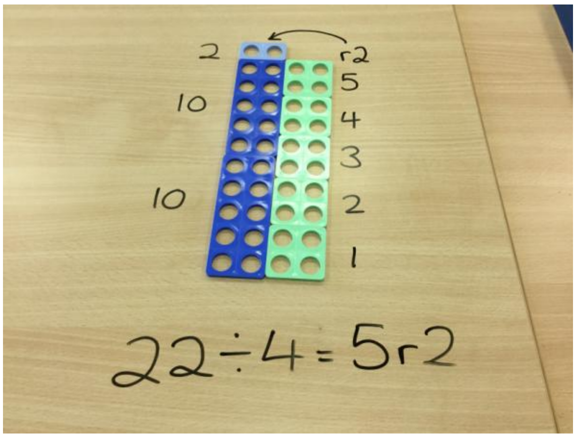
A3 Photo examples of Draw it/Make it

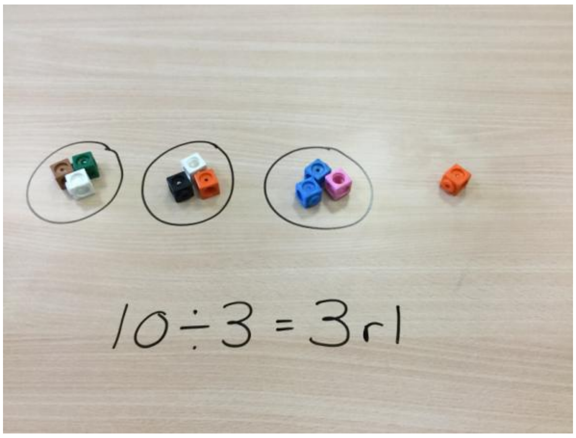
Examples for multiplication and division:

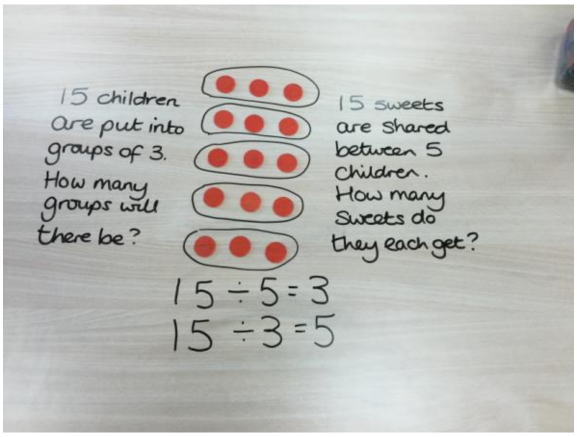


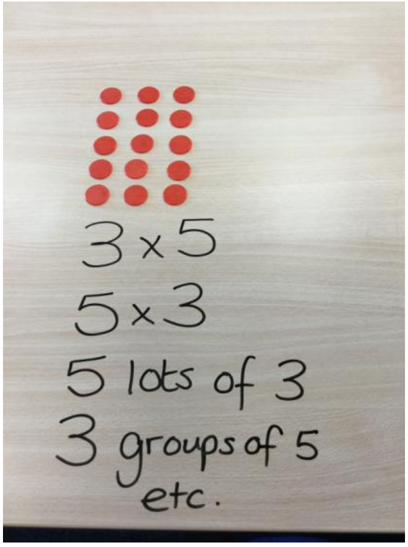


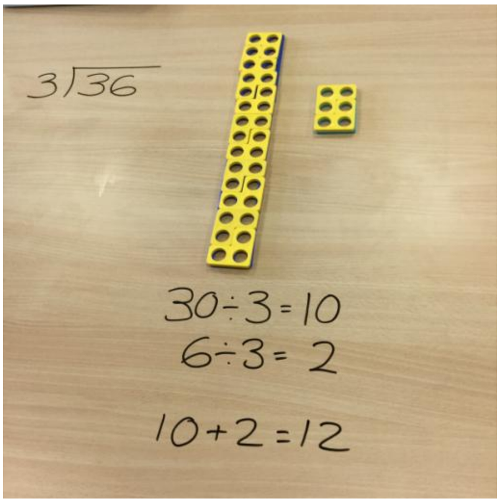


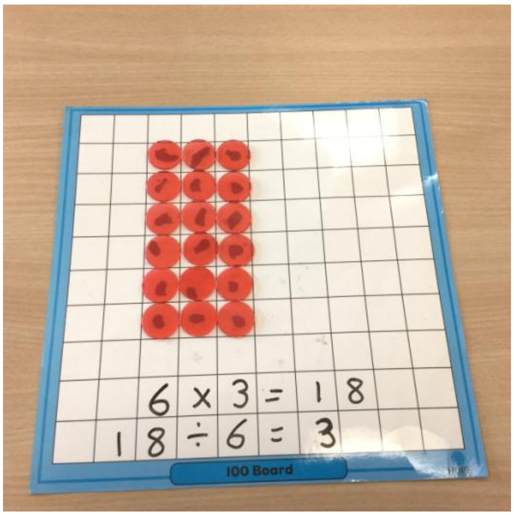


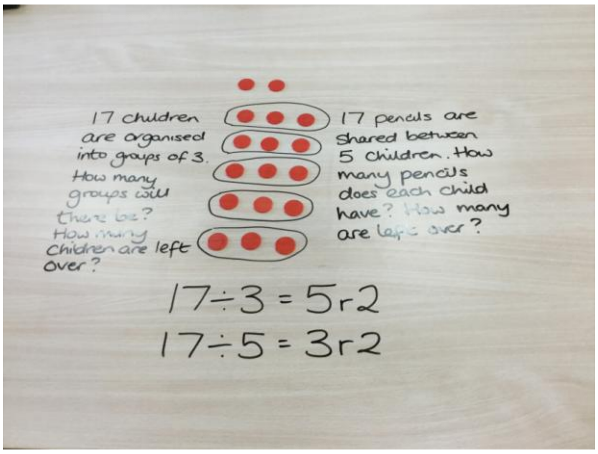


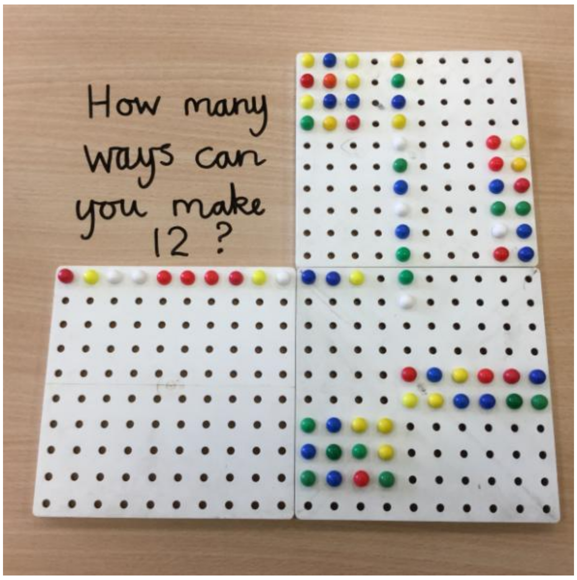


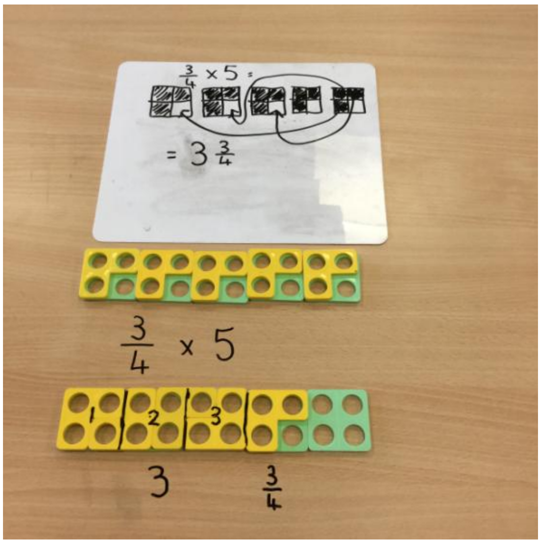

3 × 5
5 × 3
5 lots of 3
3 groups of 5
etc.

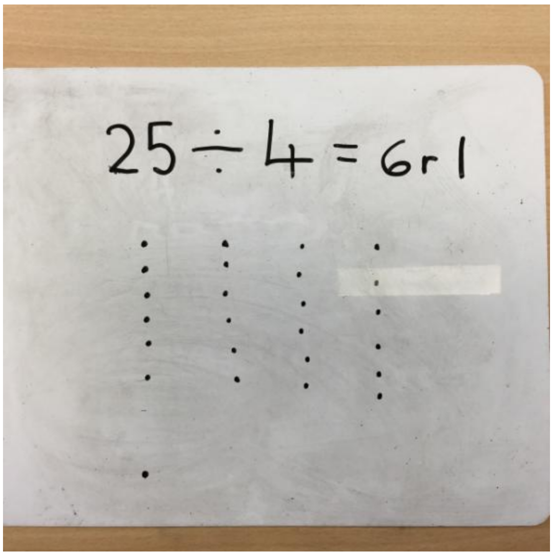

3⟌36
30 ÷ 3 = 10
6 ÷ 3 = 2
10 + 2 = 12

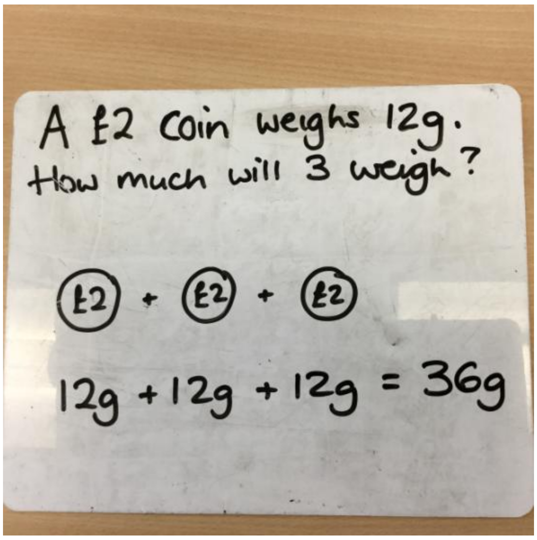

6 × 3 = 18
18 ÷ 6 = 3
100 Board

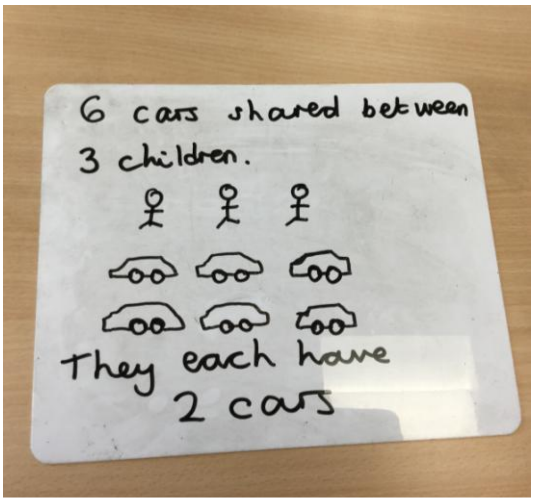

17 children are organised into groups of 3. How many groups will there be? How many children are left over?
17 pencils are shared between 5 children. How many pencils does each child have? How many are left over?
17 ÷ 3 = 5r2
17 ÷ 5 = 3r2

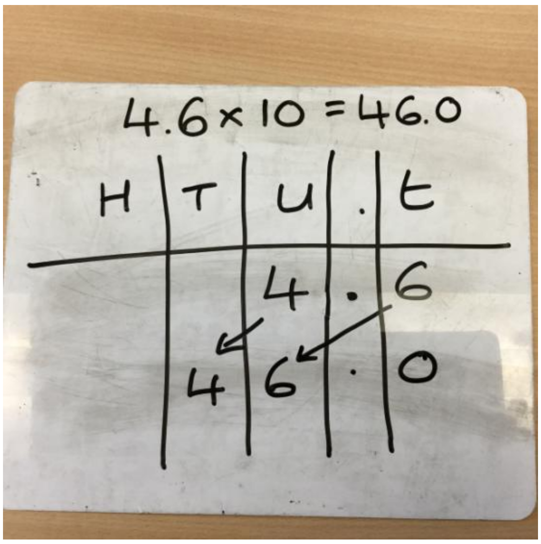

How many ways can you make 12 ?


3/4 x 5 =
= 3 3/4
3/4 x 5
1 2 3
3 3/4


25 ÷ 4 = 6r1


A £2 coin weighs 12g.
How much will 3 weigh?
£2 + £2 + £2
12g + 12g + 12g = 36g


6 cars shared between 3 children.
They each have 2 cars


4.6 x 10 = 46.0
H T U . t
4 . 6
4 6 . 0

A4 Vocabulary list

Words and phrases related to multiplication and division

Multiply
Multiplication
Multiple
Times
'Groups of'
'Sets of'
'lots of'
Product

Division
Divide
Divisible by
Rows
Columns
Lines

Prime numbers
Squared numbers
Cubed numbers

Factors
Prime factors

Inverse
Related division and multiplication calculations